

# Monte Carlo Analysis as a Trajectory Design Driver for the TESS Mission

Craig Nickel<sup>1</sup>, Ryan Lebois<sup>2</sup> and Stephen Lutz<sup>3</sup>  
*Applied Defense Solutions Inc., Columbia MD*

Donald Dichmann<sup>4</sup> and Joel Parker<sup>5</sup>  
*NASA Goddard Space Flight Center, Greenbelt MD*

**The Transiting Exoplanet Survey Satellite (TESS) will be injected into a highly eccentric Earth orbit and fly 3.5 phasing loops followed by a lunar flyby to enter a mission orbit with lunar 2:1 resonance. Through the phasing loops and mission orbit, the trajectory is significantly affected by lunar and solar gravity. We have developed a trajectory design to achieve the mission orbit and meet mission constraints, including eclipse avoidance and a 30-year geostationary orbit avoidance requirement. A parallelized Monte Carlo simulation was performed to validate the trajectory after injecting common perturbations, including launch dispersions, orbit determination errors, and maneuver execution errors. The Monte Carlo analysis helped identify mission risks and is used in the trajectory selection process.**

## Nomenclature

<i>DV</i>	=	delta-V
<i>PAM</i>	=	Period Adjustment Maneuver
<i>PLEP</i>	=	Post Lunar Encounter Perigee
<i>TCM</i>	=	Trajectory Correction Maneuver
<i>TESS</i>	=	Transiting Exoplanet Survey Satellite
<i>TLI</i>	=	Translunar Injection

## I. Introduction

In spacecraft mission design we need to be confident that there is sufficient delta-V (DV) to cope with launch vehicle injection error, maneuver execution error and orbit determination error. For the TESS mission it is a requirement that the total delta-V (deterministic plus statistical) be less than 215 m/s in 99% of cases. In fact we are required to generate a feasible trajectory that meets this ‘DV99’ requirement, and other constraints, for at least 5 dates in each month. The TESS trajectory is significantly perturbed by the gravity of the Moon and Sun through the phasing loops, the lunar flyby and the mission orbit. Moreover the allowed injection error can change the nominal 6-day orbit period of the first phasing loop by as much as one day. Consequently, the analysis to demonstrate that we can meet the DV99 constraint presents a significant challenge and it has driven some of our design choices.

In order to achieve at least 5 launch dates per month, subject to numerous constraints, we found it necessary to automate the trajectory design process as much as possible so we can examine dozens of possible solutions in each

---

<sup>1</sup> Astrodynamics Engineer, Applied Defense Solutions Inc., 10440 Little Patuxent Parkway Suite 600, Columbia, MD 21044, [CNickel@applieddefense.com](mailto:CNickel@applieddefense.com)

<sup>2</sup> Aerospace Engineer, Applied Defense Solutions Inc., [RLebois@applieddefense.com](mailto:RLebois@applieddefense.com)

<sup>3</sup> Aerospace Engineer, Applied Defense Solutions Inc., [Stephen.Lutz@applieddefense.com](mailto:Stephen.Lutz@applieddefense.com)

<sup>4</sup> Senior Navigation Engineer, NASA Goddard Space Flight Center, 8800 Greenbelt Road, Greenbelt, MD 20771, [Donald.J.Dichmann@nasa.gov](mailto:Donald.J.Dichmann@nasa.gov)

<sup>5</sup> Aerospace Engineer, NASA Goddard Space Flight Center, [Joel.J.K.Parker@nasa.gov](mailto:Joel.J.K.Parker@nasa.gov)

month. The work described in this paper is a companion to Ref. 1 in which we describe the orbit dynamics models used to design the nominal trajectories.

Like the trajectory design process, for the DV99 analysis we need a Monte Carlo simulation that largely automates the simulation and statistical analysis for the dozen of nominal trajectories examined in a month. For the Monte Carlo to function autonomously requires the development of some robust maneuver planning algorithms to cope with a variety of possible perturbations.

Our goal in this paper is to describe the analysis and simulation process developed for the Monte Carlo simulation. To meet the DV99 requirement with the given delta-V budget we found it necessary to develop maneuver planning algorithms tailored to the multi-body dynamics that TESS experiences. In fact, to deal with the variety of perturbations TESS may experience, we found it useful to develop more than one maneuver planning algorithm, each focused on a different set of desirable qualities. Then for a given set of perturbations we can try each planning method, and choose the one that requires the least delta-V while meeting mission requirements. We typically perform 400 Monte Carlo trials for each nominal trajectory to assess the DV99 value. This is a sufficient number of trials to decide whether the statistical delta-V is below 215 m/s.

The structure of the Monte Carlo simulation is based on one used for the James Webb Space Telescope<sup>2</sup>. The Monte Carlo simulation is implemented in the General Mission Analysis Tool (GMAT)<sup>3</sup>, using Matlab as a driver. To be able to analyze dozens of trajectories in a month, over a year of possible launch dates, the simulation process was automated. The Matlab driver accesses each nominal trajectory generated by the trajectory design process, performs the Monte Carlo simulation and gathers statistics. In fact, the software architecture is structured so that a single Matlab driver manages the Trajectory Design, Monte Carlo simulation and various other analyses such as correction for missed and partial burns.

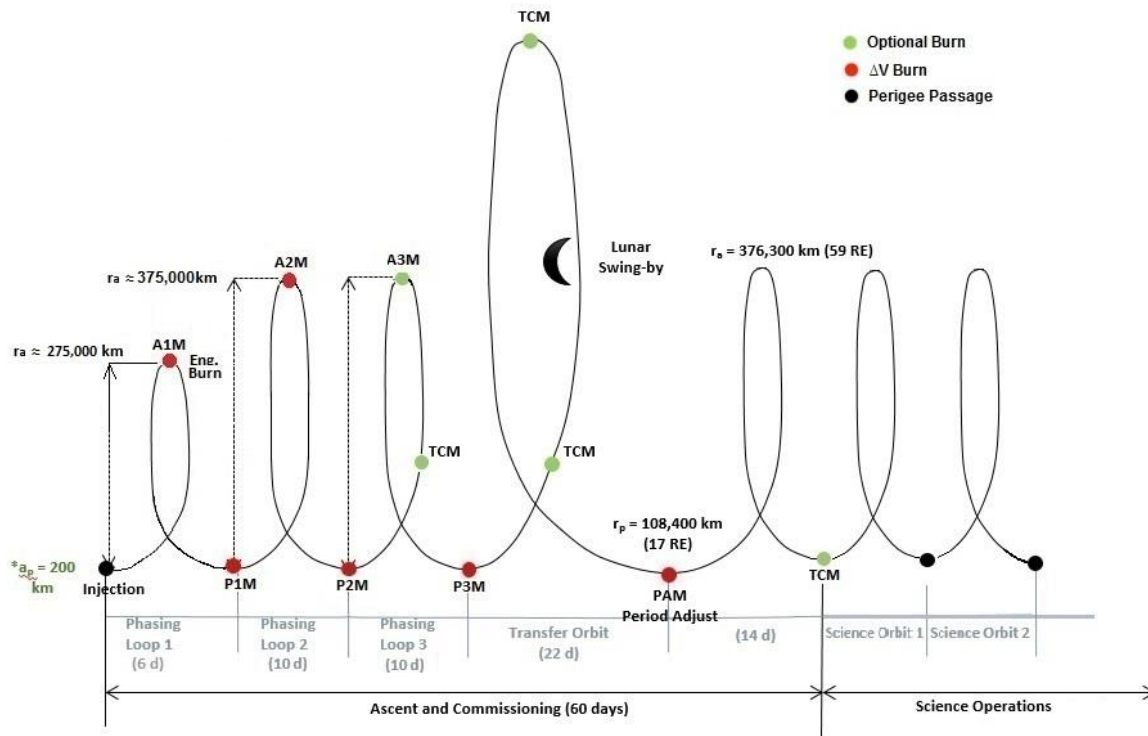
The remainder of the paper is structured as follows. In Section II we give a brief overview of the TESS mission. For more details, see the companion paper<sup>1</sup>. In Section III we discuss the modeling of the orbit errors: launch vehicle injection, maneuver execution and orbit determination. In Section IV we discuss the algorithms used to plan each maneuver, to correct for perturbations. In Section V we describe the automation used to facilitate the Monte Carlo simulation and analysis. In Section VI we summarize the MC results for launch dates in the month of January 2018. This complements the analysis of deterministic delta-V for nominal solutions described in Ref. 1. In Section VII we summarize the results in this paper, and discuss possible future work.

## II. TESS Mission Overview

The TESS mission orbit is a highly eccentric Earth orbit in 2:1 resonance with the Moon. The spacecraft will be launched from Kennedy Space Center on a SpaceX Falcon 9. Launch is planned for Dec 2017. To prepare for possible launch delays, in this paper we consider launch dates in Jan 2018.

To achieve the mission orbit perigee radius and spatial orientation we use a lunar flyby. A direct Translunar Injection (TLI) would be impractical since we would not have sufficient time and delta-V to correct for the injection error, and we would need to perform the correction with an uncalibrated engine. Instead we use 3.5 phasing loops prior to flyby, allowing us time to calibrate the engine and correct for injection error. See Figure 1. There are five nominal maneuvers in the phasing loops: A1M (at apogee A1), P1M (at perigee P1), A2M, P2M and P3M. Following the flyby, at Post Lunar Encounter Perigee (PLEP) we perform one final maneuver called the Period Adjust Maneuver (PAM) to lower apogee and achieve the 2:1 resonant orbit period of  $27.3/2 = 13.65$  days.

The Falcon 9 will place TESS in a highly eccentric orbit with perigee altitude of 200 km and apogee radius of about 275,000 km. At first apogee A1, the maneuver A1M will be performed. The maneuver A1M will have a magnitude of about 4 m/s and will be used to calibrate the engine. In early phasing loop designs it was found that the Moon gravity perturbation could lower perigee from 200 km and cause the spacecraft to enter the atmosphere if a corrective maneuver was not performed at A1M. However A1M was changed to be a calibration maneuver, and the project did not want to rely upon that small maneuver for the perigee raise. Instead, the phasing loop duration is selected so the Moon will be ahead of the spacecraft near A1 and so provide a gravity assist that raises perigee above 600 km altitude<sup>1</sup>.



**Figure 1. Phasing loop diagram for the TESS mission. The apogee for for apogees A2 and A3 is approximate, and in reality will be near the lunar orbit radius at flyby.**

At the perigee after injection, the maneuver P1M is used to raise apogee. At second perigee the maneuver P2M is used to raise apogee, if necessary, and to adjust the timing. One important finding in the analysis of the effects of maneuver errors is that it can be very costly in delta-V to correct for an error in P2M. The reason is, we need to arrive at the lunar flyby near the planned time to achieve the planned effect. If maneuver P2M is very different from the planned value then the period of loop 3 is changed significantly. There is nowhere in loop 3 before the next perigee P3 to efficiently adjust orbit period, so a large adjustment in P3M may be needed to correct the timing error. Likewise it can be very costly in delta-V to correct for an error in P3M. Therefore to reduce mission risk and to reduce statistical delta-V, whenever practical we design the phasing loops so that apogee radius at A2 and A3 is close to the lunar orbit radius at flyby. As a result, maneuver P2M and P3M are typically near zero. During the phasing loops, lunisolar perturbations can cause perigee altitude to drop below 600 km if not corrected. We use the A2M maneuver primarily to keep perigee at P2 and P3 above 600 km.

Following the flyby TESS enters the transfer orbit. The transfer orbit is in the orbit plane needed for the mission orbit, and the transfer orbit has the required orbit radius. The apogee radius of the transfer orbit is designed to achieve special timing: When TESS reaches PLEP, the spacecraft is nearly along the Earth-Moon line.

Because the spacecraft and the Moon are nearly aligned at PLEP, and the spacecraft is in 2:1 resonance with the Moon, at each subsequent spacecraft apogee the angular separation between the spacecraft and the Moon is large. This Lunar Resonant Phasing condition provided long-term stability for the orbit and eliminates the need to orbit maintenance maneuvers during the mission lifetime<sup>1,4</sup>.

### III. Orbit Error Modeling

In the Monte Carlo simulation we model three types of errors: launch vehicle injection, maneuver execution and orbit determination errors.

Launch vehicle injection error is modeled based on an injection error covariance provided by SpaceX. The covariance was delivered in the Radius-Tangent-Normal frame. For each trajectory we transform the covariance into the J2000 frame based on the nominal injection state. The statistics for injection errors allowed for the TESS mission are shown in Table 1. The allowed error in apogee radius of 30,000 km (3 sigma) translates to an error in the first phasing loop orbit of up to 1 day of the nominal 6-day period.

Orbit element	Allowed error (3 sigma)
Apogee radius (km)	30,000
Perigee radius (km)	15
Inclination (deg)	0.1
Right Ascension of Ascending Node (deg)	0.3
Argument of Perigee (deg)	0.3

**Table 1. Allowed injection error for TESS mission<sup>6</sup>.**

Maneuver execution error is modeled in terms of both magnitude and pointing error. For the initial A1M calibration maneuver the magnitude error is assumed to be 5% (3 sigma). After engine calibration, the magnitude error is assumed to be 1% (3 sigma). These values are based on historic performance. Pointing error is based on data provided by the spacecraft manufacturer, Orbit ATK. As with the magnitude error, the pointing error is expected to decrease following calibration at A1M.

Orbit determination error was determined by the TESS Flight Dynamics Team using the Orbit Determination Tool Kit developed by Analytical Graphics Inc. The orbit determination error statistics are listed in Table 2.

Maneuver	3-Sigma Position Error (km)	3-Sigma Velocity Error (m/s)
A1M, A2M	2	0.02
P1M	7	0.33
P2M, P3M	2	0.03
PAM	1	0.02

**Table 2. Orbit determination errors are maneuvers.**

At the start of the Monte Carlo simulation we generate enough Gaussian random numbers for 400 trials. The random draws are based on a seed, so the same random numbers are produced each time a Monte Carlo simulation is run. The resulting reproducibility of the trials makes it much easier to debug the maneuver planning algorithms during development. It also makes it much easier to compare different maneuver planning algorithms.

### IV. Maneuver Planning Algorithms

In this section we summarize the algorithms used to replan maneuvers following maneuver errors.

A key driver for maneuver planning in the phasing loops is the need to perform the lunar flyby very close to the planned epoch in order to achieve the transfer orbit that we use to place us in the mission orbit. At the epoch of the flyby we also need to achieve nearly the same B-plane parameters. However if the flyby epoch is different from the nominal value, or if the TLI state is different from the nominal value, then the V-infinity vector will differ from the nominal. As a consequence, even if we achieve the nominal B-plane parameters then we will not achieve the same flyby. So while we see the B-plane parameters are a valuable guide to how to reach the mission orbit, ultimately we target on the mission orbit Keplerian elements to achieve the mission orbit.

A common way to correct for maneuver error would be to use a Trajectory Correction Maneuver (TCM) to return to the nominal trajectory, performed a certain time interval - typically a few days - after the previous maneuver. In the context of Keplerian motion, this is achieved with a Lambert solution that requires two maneuvers: The first maneuver is designed to achieve the required position after a specified time interval. The second maneuver achieves the required velocity.

A clear advantage of the Lambert solution is that it returns the spacecraft to the nominal trajectory, and the subsequent maneuvers do not need to be replanned to achieve the mission orbit. The Lambert solution is the foundation for the statistical DV tool being developed jointly by NASA Goodard Space Flight Center and Johns Hopkins University Applied Physics Laboratory<sup>5</sup>.

However for the TESS phasing loops it is not always practical to employ the full force model Lambert solution. This is because the Lambert solution assumes two-body motion, but throughout its trajectory the TESS spacecraft is significantly affected by the gravity of the Moon and Sun in addition to the Earth. So we needed to develop a different approach that allows for the multibody dynamics. As noted above, the focus is on achieving the flyby close to the nominal epoch. Beyond that, there are various options for how to replan the maneuver sequence. For that reason we found it useful to develop several variations on the maneuver planning algorithm. We currently use two methods. For each Monte Carlo trial we can try each maneuver planning algorithm and choose the solution that achieves the required mission orbit and requires the least total delta-V. Below we describe our primary version of the maneuver planning algorithm, which formed the basis for the other method we use.

An injection error can cause an error in the period of the first phasing loop, as well as inclination and Argument of Perigee. The error statistics are given in Table 1. The TESS project does not plan to perform a TCM before or at maneuver A1M. This is partly because there may not be sufficient time in the operational timeline to replan and validate A1M after injection, and partly because the engine is not calibrated until after A1M. The first opportunity to efficiently correct for injection error is at or near the first perigee maneuver, P1M. Consequently, we choose to replan maneuvers at P1M and P2M to reach P3 at the nominal epoch. Specifically, we first plan P1M to achieve perigee P2 at the nominal epoch, and replan P2M to achieve perigee P3 at the nominal epoch. We then replan P1M and P2M together to achieve the nominal P3 epoch and to minimize the sum of P1M and P2M magnitudes.

At P3, we use a two-step process to achieve the desired flyby.

1. (Coarse) Vary maneuver P3M to achieve the nominal B-plane parameters to within an accuracy of 100 km.
2. (Fine) Vary P3M to achieve the transfer orbit apogee radius and perigee radius.

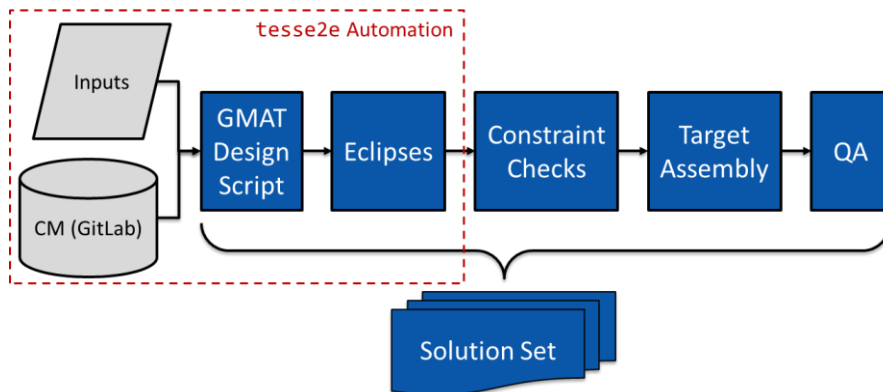
Finally, the PAM is replanned to achieve the ideal resonant mission orbit period of 13.65 days.

The process to correct for errors after injection is based upon the process described above. In general we only replan the nominal maneuvers A1M, P1M, A2M, P2M, P3M and PAM, with some exceptions. Following P2M we can insert the first TCM, labelled TCM1, about one day after P2M. Following P3M we can insert the second TCM, labelled TCM2, about one day after P3M. After the flyby, we can also plan a TCM at the Post Lunar Encounter Apogee. Following PAM we can insert the TCM3, about a day later. We can also perform a final TCM, labelled PAM2, at the perigee after PAM.

The secondary maneuver planning algorithm uses the primary algorithm but does minimize delta-V when targeting the Lunar B-plane parameters and the transfer orbit. Although this algorithm results in larger delta-V costs, most cases are still within the 215 m/s mission delta-V budget. The advantage of this secondary algorithm is that it allows for convergence of more solutions. A merged set of solutions from both algorithms defines the MC results for a given launch date.

## **V. Automation of Trajectory Design and Analysis**

The integrated trajectory design and analysis software developed for TESS automates the process of nominal trajectory design followed by the Monte Carlo simulation and other analyses. The process to generate nominal solutions is represented in Figure 2. Details on the nominal trajectory design algorithm are in Ref. 1.

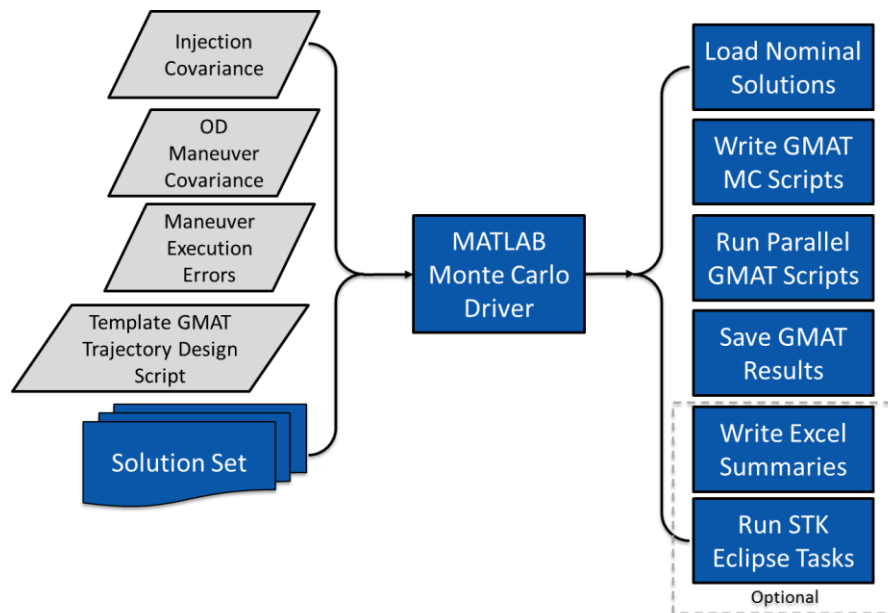


**Figure 2. TESS Nominal Trajectory Design Process.** Spacecraft parameters, error statistics and force model data are stored in a Configuration Managed (CM) database in Gitlab. Those data plus simulation control parameters serve as inputs to the GMAT design script. After completion of the GMAT script, the architecture performs eclipse analysis and other constraint checking. This is followed by target assembly to select a solution for each feasible launch date. Finally, Quality Assurance (QA) is performed on the solutions to confirm that requirements are satisfied.

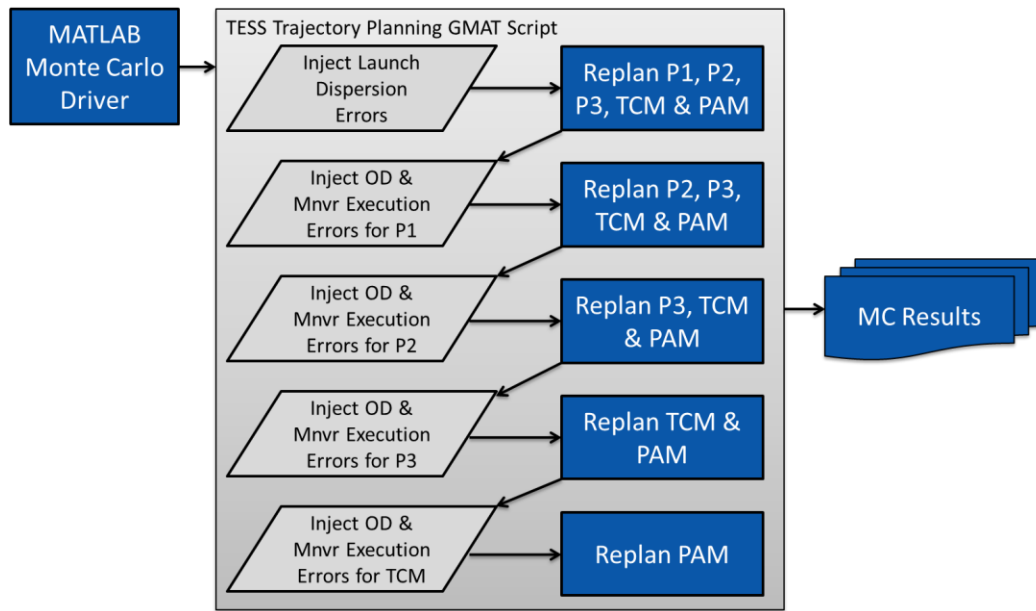
The Monte Carlo simulation takes several input parameters. First, it requires a nominal trajectory, including a launch vehicle separation state vector and delta-Vs for all of the maneuvers. The Monte Carlo simulation can handle as many launch dates to analyze as the operator desires. It is common for us to run the Monte Carlo simulation over an entire month's worth of nominal trajectory solutions, which typically contains about 20 solutions.

Secondly, the simulation requires the error statistics described in Section III. Finally, the simulation requires a template GMAT scenario file (\*.script file), which contains the forward-looking maneuver planning algorithms. The forward-looking maneuver planning algorithms used for the Monte Carlo simulation are distinct from those used in the nominal trajectory design, which employ multiple shooting<sup>1</sup>. The template GMAT script replans all subsequent maneuvers after executing each maneuver with its associated perturbations.

The Monte Carlo simulation uses Matlab to execute a series of processes to ingest the inputs, generate the random draws for all of the perturbations, generate the GMAT scenario files (\*.script files), run parallel instances of GMAT, retrieve and manage the run results, and then execute optional post-processing procedures, including eclipse constraint checking. The Monte Carlo architecture is shown in Figure 3.



**Figure 3. Monte Carlo Flow diagram.**



**Figure 4. Structure of the TESS Maneuver Planning GMAT script**

The implementation of the maneuver sequence of maneuver planning algorithms is shown in Figure 4.

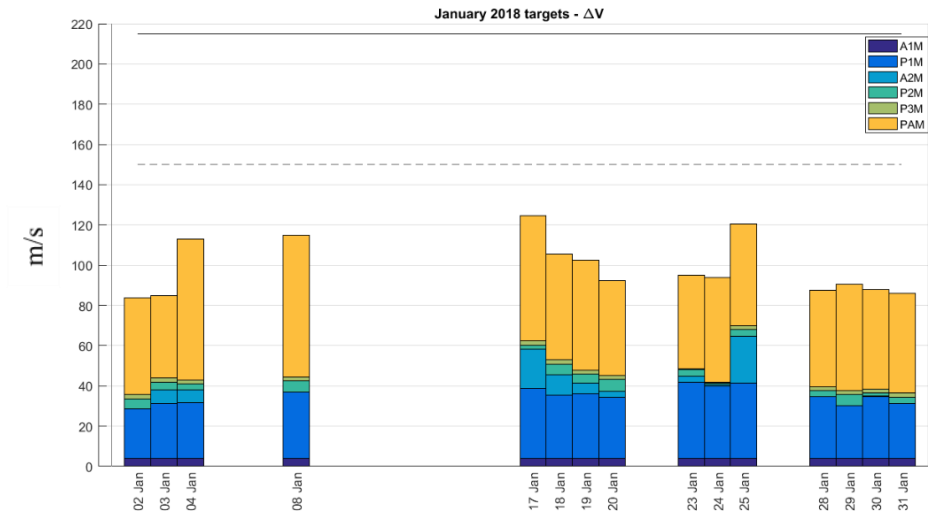
Processing time becomes the major constraint when bounding the Monte Carlo simulation. Each Monte Carlo trial takes between 5 and 15 minutes to converge and produce results. A sufficient number of random trials per solution is needed to generate statistically meaningful results. We run 400 trials, which produces an accuracy of about  $1/\sqrt{400} = 5\%$ . Because the statistical delta-V values are 60 m/s or less, this means the uncertainty is about  $0.05 \cdot 60 = 3$  m/s. So if the total delta-V we find is no more than  $215 - 3 = 212$  m/s, we can be confident that the DV99 value is below 215 m/s.

At least one nominal solution must be assessed for each potential launch date. Using Matlab to drive the process instead of looping within GMAT (as it was performed initially) allows the team to parallelize the process. We currently use 10-15 parallel processes per machine, with plans to increase that number. Parallelization reduces processing time for a single nominal solution from ~4 days to less than 10 hours. We can verify that each launch opportunity will be within total delta-V allocation and will meet the eclipse constraints. Moreover, we can use the Monte Carlo results as part of the trajectory selection process, to identify the feasible launch opportunities that will have the least total (deterministic plus statistical) delta-V.

The Matlab driver code was adapted for use with nominal trajectory design planning and maneuver contingency planning. The inputs to each process differed from those of the Monte Carlo, but the underlying Matlab code that allows for parallel processing remained the same. The trajectory design driver requires a launch date and a maximum phasing loop duration. The maneuver contingency planning driver specifies scale factors for each maneuver: A1M, P1M, A2M, P2M, P3M, and PAM. The maneuver contingency planning driver allows us to model missed and partial burn cases. The driver first runs a specified Monte Carlo draw to obtain a baseline delta-V. Then, using user-supplied maneuver scale factors, uses the Monte Carlo algorithm to plan and replan the mission trajectory based on the missed or partial burns.

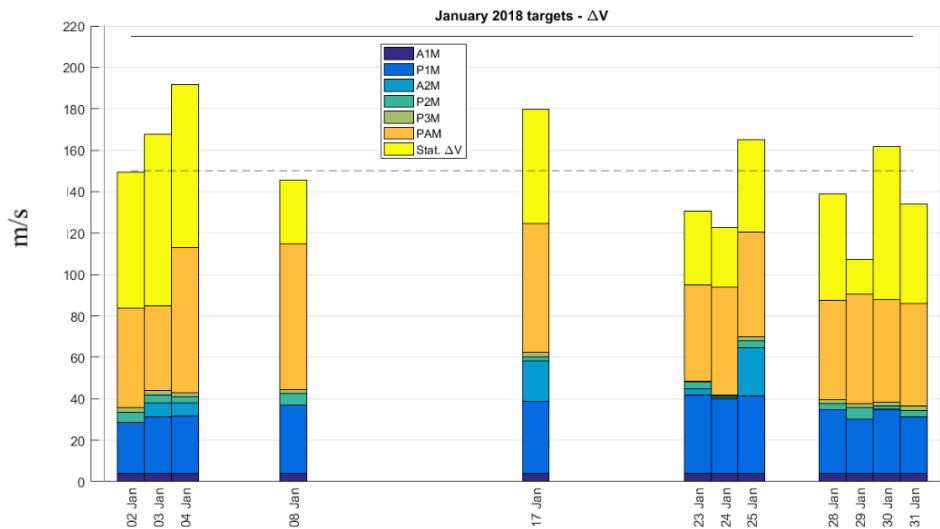
## VI. Summary of Monte Carlo Results

In Ref. 1 we describe the results of TESS trajectory design for launches in December 2017 through February 2018. Figure 5 below reproduces Figure 17 in Ref. 1, showing the stacked bar chart of delta-V for launches in January 2018.



**Figure 5. Stacked bar chart of delta-V for each feasible solution in January, prior to DV99 evaluation.**

Figure 6 extends the data in Figure 5 to include the statistical delta-V. We found that the DV99 constraint was met for 12 of the 15 solutions found to be feasible prior to the statistical delta-V analysis. We did not get a sufficient number of the 400 trials to converge for launches on January 18, 19 and 20 to pass the DV99 test. It is possible that, with the use of another maneuver planning algorithm, January 18-20 could also be qualified for launch. For the 12 dates that do meet the DV99 constraint, the total delta-V including statistical delta-V is below 190 m/s.



**Figure 6. Stacked bar chart for DV for each feasible solution in January, including statistical delta-V. Solutions for launch dates Jan 18 – 20 were not confirmed as feasible.**



## VII. Conclusion

A statistical delta-V analysis for the TESS mission presents special challenges. We needed to develop maneuver planning algorithms tailored to the multibody dynamics experienced on the trajectory, and the algorithms must be able to handle injection errors that could alter the maneuver timeline by a day. We employ multiple planning algorithms to achieve greater flexibility and more converged cases in the Monte Carlo simulation. We plan to use these same maneuver planning algorithms in flight operations, so the hundreds of trials in the Monte Carlo simulation demonstrate the robustness of the algorithms.

To handle a large number of nominal trajectories and a large number of Monte Carlo trials, a unified software architecture was developed using GMAT and Matlab to manage trajectory design, Monte Carlo simulations and other orbit analyses. Our simulation results for January 2018 launches show that we can meet the requirement of DV99 requirement for at least five launch dates. In fact we have at least 12 launch dates in January from which the project can select to find the best mission opportunities.

In the near future we expect to receive further launch vehicle injection covariances for SpaceX, after they complete their own Monte Carlo simulations. We plan to perform further Monte Carlo simulations for launch dates in the remainder of 2018.

## Acknowledgments

The authors thank Chad Mendelsohn and Alinda Mashiku for valuable comments. The fourth author (D.D.) thanks Justin Atchison and Fazle Siddique of the Johns Hopkins University Applied Physics Laboratory for valuable insights on statistical delta-V analysis, in the context of a joint DV99 research and development project.

## References

<sup>1</sup>Dichmann, D.J., Parker, J.J., Nickel, C., Lutz, S., “Trajectory Design Enhancements to Mitigate Risk for the Transiting Exoplanet Survey Satellite (TESS). AIAA/AAS Astrodynamics Specialist Conference, Long Beach CA, Sept. 2016.

<sup>2</sup>Dichmann, D.J., Alberding, C.M., Yu, W.H. “Stationkeeping Monte Carlo Simulation for the James Webb Space Telescope”. International Symposium on Space Flight Dynamics, Laurel MD, May 2014.

<sup>3</sup>General Mission Analysis Tool, NASA Goddard Space Flight Center, version R2015a, <http://gmatacentral.org>.

<sup>4</sup>Dichmann, D.J., Parker, J.J., Mendelsohn, C.M. “Trajectory Design for the Transiting Exoplanet Survey Satellite. International Symposium on Space Flight Dynamics”, Laurel MD, May 2014.

<sup>5</sup>NASA Techport website for DV99, <https://techport.nasa.gov/view/40605>

<sup>6</sup>Benson, W., Jones, T., Engelhardt, R. “TESS Trajectory Analysis Input Specifications (PGAA-1)”, ELVL-2015-0043923 Revision A, Feb 2015.